Grauert theory and recent complex geometry[1]
(Grauert 理論と最近の複素幾何)

February 6th.

15:00–16:00 Ken-ichi Yoshikawa (Kyoto University)
Degeneration of Riemann surfaces and small eigenvalues of Laplacian

16:10–17:10 Shin-ichi Matsumura (Tohoku University)
On asymptotic base loci of relative anti-canonical divisors of algebraic fiber spaces

17:40–18:40 Masataka Iwai (Kyoto University, Osaka City University)
On projective manifolds whose tangent bundles contain positive subbundles

20:30–21:30 Daniel Greb (Universität Duisburg-Essen)
Projectively flat klt varieties

February 7th.

15:00–16:00 Hajime Tsuji (Sophia University)
Invariant metrics associated with L^p-holomorphic functions

16:10–17:10 Genki Hosono (Tohoku University)
Variational theories in L^2 and pluripotential theories

17:40–18:40 Yusaku Tiba (Ochanomizu University)
Cohomology of vector bundles and non-pluriharmonic loci

20:30–21:30 Tomoyuki Hisamoto (Tokyo Metropolitan University)
Geometric flow, Multiplier ideal sheaves, and optimal degeneration of a Fano manifold

February 8th.

15:00–16:00 Tohru Morimoto (Seki Kowa Institute of Mathematics and Institut Kiyoshi Oka de Mathématiques)
Does a formal equivalence imply an analytic equivalence for geometric structures?

16:10–17:10 Takayuki Koike (Osaka City University)
Linearization of transition functions of a semi-positive line bundle along a certain submanifold

17:40–18:40 Masanori Adachi (Shizuoka University)
On Levi flat hypersurfaces with transversely affine foliation

20:30–21:30 Takao Akahori (Hyogo University)
Deformations and CR invariants

February 9th.

15:00–16:00 Kazuko Matsumoto (Tokyo University of Science)
Derivatives of Hartogs q-pseudoconcave sets and essential singularities of analytic sets

16:10–17:10 Thomas Pawlaschyk (University of Wuppertal)
Foliations of continuous q-pseudoconcave graphs

17:40–18:40 Yuta Kusakabe (Kyoto University)
The Oka principle and the dual Levi problem

18:50–19:50 Martin Sera (Kyoto University of Advanced Science)
On a mixed Monge-Amp̀re operator for quasiplurisubharmonic functions

[1]This conference is supported by Osaka City University Advanced Mathematical Institute: MEXT Joint Usage/Research Center on Mathematics and Theoretical Physics (文科省共同利用・共同研究).

Titles and Abstracts

On Levi flat hypersurfaces with transversely affine foliation

Masanori Adachi (Shizuoka University)

In this talk, we discuss the classification problem of Levi flat hypersurfaces in complex surfaces by restricting ourselves to the case that the Levi foliation is transversely affine. After presenting known examples, we give a proof for the non-existence of real analytic Levi flat hypersurface whose complement is 1-convex and Levi foliation is transversely affine in a compact Kähler surface. Note that such an example does exist in Inoue-Hirzebruch surfaces. This is a joint work with Séverine Biard.

Projectively flat klt varieties

Daniel Greb (Universität Duisburg-Essen)

In the context of uniformisation problems, I will study projective varieties with klt singularities whose cotangent sheaf admits a projectively flat structure over the smooth locus. I will present joint work with Stefan Kebekus and Thomas Peternell, in which we prove that torus quotients are the only klt varieties with semistable cotangent sheaf and extremal Chern classes. This generalises an earlier result of Jahnke and Radloff. In the talk, I will concentrate on the complex-analytic aspects of the proof.

Geometric flow, Multiplier ideal sheaves, and optimal degeneration of a Fano manifold

Tomoyuki Hisamoto (Tokyo Metropolitan University)

In 2005 Donaldson showed that the Calabi functional is bounded from below by the supremum of normalized Donaldson-Futaki invariants. For Fano manifolds we reformulate the problem in terms of the Ricci potential and show that the lower bound precisely equals to the supremum of normalized Ding invariants. The proof introduces the new geometric flow and the test configurations are constructed by taking multiplier ideal sheaves of the flow.

Variational theories in L^2 and pluripotential theories

Genki Hosono (Tohoku University)

In recent development of complex analysis of several variables, variational results take an important place. In this talk, I will explain some of these results, such as relations between variation of Bergman kernels and L^2-extension theories and pluripotential theoretic results concerning pluripotential Green functions.

On projective manifolds whose tangent bundles contain positive subbundles

Masataka Iwai (Kyoto University, Osaka City University)

If the tangent bundles of projective varieties are "positive"(such as ample, nef, and so on), we have the structure theorems of the projective varieties. On the other hand, Peternell proposed problems on the structures of projective manifolds whose tangent bundles contain "positive" subbundles. In this talk, I will talk about the related researches and my results on Peternell's problems.

Linearization of transition functions of a semi-positive line bundle along a certain submanifold

Takayuki Koike (Osaka City University)

Let X be a complex manifold and L be a holomorphic line bundle on X. Assume that L is semi-positive, namely L admits a smooth Hermitian metric with semi-positive Chern curvature. Let Y be a compact Kähler submanifold of X such that the restriction of L to Y is topologically trivial. We investigate the obstruction for L to be unitary flat on a neighborhood of Y in X. As an application, we show the existence of nef, big, and non semi-positive line bundle on a non-singular projective surface. Such an example is constructed by considering an analogy of the Grauert's example on the non-projectivity of a variety obtained by contracting a negative curve of a projective surface.

The Oka principle and the dual Levi problem
Yuta Kusakabe (Kyoto University)

The homotopy principle in complex analysis is called the Oka principle. Grauert, Gromov, Forstnerič and others developed it into the theory of Oka manifolds which are dual to Stein manifolds. On the other hand, the Levi problem asks for the geometric characterization of Stein domains. As is well known, Grauert also solved the Levi problem on complex manifolds. In this talk, we consider the "dual" Levi problem which asks for the geometric characterization of Oka domains.

Derivatives of Hartogs q-pseudoconcave sets and essential singularities of analytic sets
Kazuko Matsumoto (Tokyo University of Science)

Let D be an open subset in $\mathbb{C}^n$, let A be an analytic subset of pure dimension k in D, and let S be an analytic subset of pure dimension q in $D \setminus A$. In the 1060s, K. Oka presented to his students the following problems:

Problem 1. The strong (or weak) derivative of a Hartogs q-pseudoconcave set in D is also Hartogs q-pseudoconcave in D.

Problem 2. If $k > q$, the set of essential singulalities of $A' \subset S$ is Hartogs q-pseudoconcave in D.

In this talk, these problems will be discussed.

On asymptotic base loci of relative anti-canonical divisors of algebraic fiber spaces
Shin-ichi Matsumura (Tohoku University)

This talk focuses on the relative anti-canonical divisor of an algebraic fiber space. I would like to discuss relations among positivity of the relative anti-canonical divisor, the certain flatness of direct image sheaves, and variants of the base loci, including the stable (augmented, restricted) base loci and upper-level sets of Lelong numbers. This talk is based on joint work with Sho Ejiri and Masataka Iwai.

Does a formal equivalence imply an analytic equivalence for geometric structures?
Tohru Morimoto (Seki Kowa Institute of Mathematics and Institut Kiyoshi Oka de Mathématiques)

In mathematics we know many interesting works done concerning the following question: of convergence:

[F to A](equation) : Does the existence of a formal solution of an equation imply that of an analytic solution?

It is well known that if the equation is an analytic (functional) equation then the question is affirmative by the theorem of M. Artin, that is [F to A](analytic equation) is yes. But if the equation includes differential equations the question is delicate. What we can say in general is t that [F to A](involutive system) is Yes. Here "involutive system" is the notion introduced by Élie Cartan in his theory now called Cartan -Kähler theorem. He studied general systems of partial differential equations as Pfaff equations and defined a system in involution geometrically in terms of Pfaff systems and proved the convergence by iterated use of the theorem of Caucy-Kowalevsky. The formal and algebraic aspects became well understood after the works of Kuranishi, Spencer, Sternberg, Quillen Goldschmidt and Serre. For the convergence an alternative elegant proof was given by Malgrange by using the privileged neighbourhood theorem which has the origin in Oka and developed by Grauert and Malgrange. Initiating "nilpotent analysis" and extending the privileged neighbourhood theorem from abelian to nilpotent, Morimoto gave a non-trivial generalization of Cartan - Kähler theorem to weightedly involutive systems of non-linear PDE's with possibly singularities.

In this talk, as indicated by the title, I like to discuss the problem [F to A](geometric structure): Does a formal equivalence imply an analytic equivalence for geometric structures. This is essentially the problem [F to A](differential equation). But the geometric method to obtain the complete invariants of a geometric structure through prolongation and reduction invented by E. Cartan and then developed by Singer-Sternberg, Tanaka, Morimoto, and recently by J. Hong and Morimoto, enables one to reduce the original differential equation for equivalence to a distinguished type of differential equation that we call quasi Lie equation. Then we discuss [F to A](quai Lie equation) by using the generalized Cartan-Kaehler theorem as a main tool.

Foliations of continuous q-pseudoconcave graphs Thomas Pawlaschyk and Nikolay Shcherbina (University of Wuppertal)

The notion of pseudoconvexity and the problem of existence of a complex structure on a given set play important roles in complex analysis. The first classical and fascinating result in this direction for graphs of real codimension two which are merely continuous is due to Hartogs [Har09] and comes back to 1909. It states that the complement of the graph of a continuous function $f : B \to \mathbb{C}$ defined on a ball in $\mathbb{C}^n$ is a domain of holomorphy if and only if the function f is holomorphic. A similar statement holds true for continuous graphs of real codimension one and was first proved in a fundamental paper of Levi from 1911 for smooth graphs in the special case $n = 1$. It was established much later in [Shc93] by Shcherbina in 1993 that if we drop the smoothness assumption on the function and only demand continuity the same results holds true. Later, based on this result, the same statement for continuous functions was proved by Chirka [Chi01] in 2001 in the case $n \geq 2$.

The main purpose of my talk is to present a generalization of these results to the case of continuous graphs of higher codimension, i.e., to the case when the target space has dimension larger than one. In this case the requirement of pseudoconvexity of the complement of the graph is not a proper condition any more. However, it turns out that if we substitute the condition of pseudoconvexity of the complement by its n-pseudocovexity in the sense of Rothstein [Rot55], then the following generalization holds true:

Let n, k, p be integers with $n \geq 1$, $p \geq 0$ and let $k \in \{0, 1\}$ such that $N = n + k + p \geq 2$. Let B be the unit ball in $\mathbb{C}^n_z \times \mathbb{R}^k_u$ and let $f : B \to \mathbb{R}^k_v \times \mathbb{C}^p_\zeta$ be a continuous map. Then the complement of its graph $\Gamma(f)$ in $B \times \mathbb{R}^k_v \times \mathbb{C}^p_\zeta \subset \mathbb{C}^N_{z,u+iv,\zeta}$ is n-pseudoconvex in the sense of Rothstein if and only if $\Gamma(f)$ is foliated by n-dimensional complex submanifolds.

This result has already been published in my doctoral thesis [Paw15] from 2015.

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On a mixed Monge-Ampr̀e operator for quasiplurisubharmonic functions
Martin Sera (Kyoto University of Advanced Science)

This reports on a joint work with R. Lärkäng and E. Wulcan. We consider mixed Monge-Ampère products of quasiplurisubharmonic functions with analytic singularities (introduced in a previous work with H. Raufi additionally). These products have the advantage that they preserve mass (a property which is missing for non-pluripolar products).

The main result of the work presented here is that such Monge-Ampère products can be regularized as explicit one parameter limits of mixed Monge-Ampère products of smooth functions, generalizing a result of Andersson-Błocki-Wulcan. We will explain how the theory of residue currents, going back to Coleff-Herrera, Passare and others, plays an important role in the proof.

As a consequence, we get an approximation of Chern and Segre currents of certain singular hermitian metrics on vector bundles by smooth forms in the corresponding Chern and Segre classes.

Cohomology of vector bundles and non-pluriharmonic loci

Yusaku Tiba (Ochanomizu University)

Let X be a Stein manifold and let $F \to X$ be a holomorphic vector bundle. Let φ be an exhaustive plurisubharmonic function on X. Here φ is upper semi-continuous function and is possibly non continuous. We denote by $\operatorname{supp} i\partial\overline{\partial}\varphi$ the support of $i\partial\overline{\partial}\varphi$. In this talk, we study a relation between the cohomology groups of F on neighborhoods of $\operatorname{supp} i\partial\overline{\partial}\varphi$ and those on X. Our main result is the following:

Theorem

Let X be a Stein manifold of dimension n ($n \geq 3$). Let m be a positive integer which satisfies $1 \leq m \leq n-2$. Let $\varphi_1, \dots, \varphi_m$ be non-constant plurisubharmonic functions on X such that for every $r < \sup_X \varphi_j$ the sublevel set $\{z \in X \,|\, \varphi_j(z) \leq r\}$ is compact ($1 \leq j \leq m$). Let F be a holomorphic vector bundle over X. Then the natural map

$$H^0(X, F) \to \varinjlim_{\bigcap_{j=1}^m \operatorname{supp} i\partial\overline{\partial}\varphi_j \subset V} H^0(V, F).$$

is an isomorphism and

$$\varinjlim_{\bigcap_{j=1}^m \operatorname{supp} i\partial\overline{\partial}\varphi_j \subset V} H^q(V, F) = 0$$

for $0 < q < n - m - 1$.

As a corollary, we give a variant of Lefschetz hyperplane theorem on a Stein manifold, that is, the low-degree cohomology groups of a neighborhood of $\operatorname{supp} i\partial\overline{\partial}\varphi$ is determined by those of X. We also show that the similar statements hold on a projective algebraic manifold.

Invariant metrics associated with L^p-holomorphic functions

Hajime Tsuji (Sophia University)

We consider a generalization of Bergman metrics associated with L^p-holomorphic functions and consider some applications.

Degeneration of Riemann surfaces and small eigenvalues of Laplacian

Ken-ichi Yoshikawa (Kyoto University)

Let $f \colon X \to S$ be a one parameter family of compact Riemann surfaces over a curve. Assume that X is a compact Kähler surface. Then the fibers of f are endowed with the metric induced from the one on X. If the singular fiber of f is not irreducible, then some eigenvalues of the Laplacian of the regular fiber converge to zero as the regular fiber approaches to the singular fiber. We call such eigenvalues small eigenvalues. Here the Laplacian means the one acting on the functions of each regular fiber. In this talk, when a singular fiber is given, I would like to determine the asymptotic behavior of the "product" of all small eigenvalues of Laplacian.

Deformations and CR invariants

Hyogo University
Takao Akahori

Abstract

Let N be a complex manifold and let M be a compact strongly pseudo convex real hyper surface in N with $dim_{\mathbf{R}} M \geq 5$. Then over M, we have a CR structure, $(M, {}^0T'')$, induced from N. This CR structure is of interest. For example, this CR structure determines ambient N. We put a contact form θ over M. Then, by the standard method as Kaehler manifolds, we have several invariants. For $u \in \Gamma(M, ({}^0T') \wedge \wedge^{(k-1)}({}^0T'')^*)$, we study whether our u makes sense as an invariant of N or not. While related with deformations of isolated singularities, deformation theory of strongly pseudo convex CR manifolds, is initiated by Kuranishi. And this theory is well developed and we have the deformation complex (for the notations, see [A1],[AGL], and in [AGL], we call it the rumin complex.)

$$\Gamma(M, \mathbf{C}) \xrightarrow{\mathcal{D}} \Gamma(M, E^1) \xrightarrow{\overline{\partial}^{(1)}} \Gamma(M, E^2)$$

We see $\mathcal{D}$. $\mathcal{D}$ is as follows.

$$\{Z_g : Z_g = g \otimes \zeta + X_g, g \text{ is a complex valued } C^\infty \text{ function}\} \xrightarrow{\overline{\partial}} \Gamma(M, E_1)$$

And for $g \in \Gamma(M, \mathbf{C})$, $\mathcal{D}g$ is deterimned by: $\mathcal{D}g = \overline{\partial}(g \otimes \zeta + X_g)$. Here X_g is the Hamiltonian vector field determined by g (for the definitions, see [AGL]).

On the other hand, $Z_g = g \otimes \zeta + X_g$ is regarded as an infinitesimal deformation of displacements of M in N, which preserves the contact form in a certain sense. So for $u \in \Gamma(M, ({}^0T') \wedge \wedge^{(k-1)}({}^0T'')^*)$, it seems natural to consider that $\mathcal{L}_{Z_g} u = 0$ means : u is an invariant of N. Therefore we would like to propose the notion of "invariance" as follows. That is to say, the differential form on M, $u \in \Gamma(M, ({}^0T') \wedge \wedge^{(k-1)}({}^0T'')^*)$, is a kind of invariants of N if

$$\mathcal{L}_{Z_g} u = 0, \text{on } \wedge^k ({}^0T'')^*.$$

where $\mathcal{L}_{Z_g}$ means the Lie derivation by Z_g.

In this paper, we study this line, and this work is in progress.

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