Subvarieties and foliations of complex projective varieties

7th–10th, February, 2022, online(zoom)[1]

Program

7th, February, 2022(Monday)

GMT 6:00-7:00 (JST 15:00-16:00)
Shin-ichi Matsumura (Tohoku University)
On projective manifolds with pseudo-effective tangent bundle

GMT 7:10-8:10 (JST 16:10-17:10)
Junchao Shentu (University of Science and Technology of China)
Kollár's conjecture via L^2 technique

GMT 8:40-9:40 (JST 17:40-18:40)
Feng Hao (KU Leuven)
π_1-small divisors and fundamental groups of varieties

GMT 11:30-12:30 (JST 20:30-21:30)
Xiaojun Wu (University of Bayreuth)
Pseudo-effective vector bundle and numerical flatness

8th, February, 2022(Tuesday)

GMT 6:00-7:00 (JST 15:00-16:00)
Jihun Yum (IBS, Center for Complex Geometry)
Limits of Bergman kernels on a tower of coverings of compact Kähler manifolds

GMT 7:10-8:10 (JST 16:10-17:10)
Hoseob Seo (Seoul National University)
On L^2 extension from singular hypersurfaces

GMT 8:40-9:40 (JST 17:40-18:40)
Félix Lequen (CY Cergy Paris Université)
Almost every K3 surface contains infinitely many linear Levi-flat hypersurfaces

GMT 11:30-12:30 (JST 20:30-21:30)
Olivier Thom (Instituto de Matemática Pura e Aplicada)
Neighborhoods of higher genus curves: formal foliations and classification

9th, February, 2022(Wednesday)

GMT 6:00-7:00 (JST 15:00-16:00)
Seungjae Lee (IBS, Center for Complex Geometry)
Symmetric differentials on complex hyperbolic space forms and its L^2 holomorphic jet extension

GMT 7:10-8:10 (JST 16:10-17:10)

~~Juanyong Wang (Chinese Academy of Sciences)~~

~~On the structure of varieties of semi-Fano type~~

GMT 8:40-9:40 (JST 17:40-18:40)

Lukas Braun (University of Freiburg)

Kähler metrics, orbifold structures, and fundamental groups on resolutions of klt spaces

10th, February, 2022(Thursday)

GMT 6:00-7:00 (JST 15:00-16:00)

Genki Hosono (Tohoku University)

On the characterization of Griffiths positivity of Hermitian metrics

GMT 7:10-8:10 (JST 16:10-17:10)

Zhiwei Wang (Beijing Normal University)

Recent progress on extension of quasiplurisubharmonic functions on Kähler manifolds

GMT 8:40-9:40 (JST 17:40-18:40)

Martin Sera (Kyoto University of Advanced Science)

A smoothness criterion for complex spaces in terms of differential forms

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On the structure of varieties of semi-Fano type

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Abstracts

7th, February, 2022(Monday)

Shin-ichi Matsumura (Tohoku University)
On projective manifolds with pseudo-effective tangent bundle

In this talk, I will introduce a structure theorem of a projective manifold with pseudo-effective tangent bundle: such a manifold admits a smooth fibration onto a flat projective manifold whose general fiber is rationally connected. If time permits, I will explain an observation of this result from the viewpoint of the Minimal Model Program. This talk is based on a joint work with Genki Hosono and Masataka Iwai.

Junchao Shentu (University of Science and Technology of China)
Kollár's conjecture via L^2 technique

In 1986, Kollár's posted several conjectures (torsion freeness, vanishings, decomposition, etc.) on the derived pushforward of the dualizing sheaf twisted by a polarizable variation of Hodge structure. These conjectures were first proved by M. Saito via his theory of mixed Hodge modules. In this talk I will explain how to give a concise proof of Kollár's conjectures via L^2 technique. This method also allows us to generalize Kollár's conjectures to the context of nonabelian Hodge theory.

Feng Hao (KU Leuven)
π_1-small divisors and fundamental groups of varieties

Lasell and Ramachandran showed that the existence of rational curves of positive self-intersection on a smooth projective surface X implies that all the finite dimensional linear representations of the fundamental group $\pi_1(X)$ are finite. In this talk, I will explain how to use some perturbation methods and nonabelian Hodge theory to generalize Lasell and Ramachandran's result to the case of π_1-small divisors on quasiprojective varieties. I will also discuss some properties on π_1-small curves on smooth projective surfaces with infinite fundamental groups.

Xiaojun Wu (University of Bayreuth)
Pseudo-effective vector bundle and numerical flatness

In this talk, I will discuss the generalisation of the notion of pseudo-effective line bundle to the higher rank case. In particular, I will talk about the proof of the following result: a strongly pseudo-effective vector bundle over a compact Kähler manifold with vanishing first Chern class is numerically trivial. The proof is based on a natural construction of closed positive current in the first Chern class called Segre current. The projective case of this result has been proven by Campana-Cao-Matsumura and Hosono-Iwai-Matsumura. As a geometric application, the tangent bundle or cotangent bundle of a Calabi-Yau manifold or a symplectic irreducible holomorphic manifold is not strongly pseudo-effective. If times permits, I will talk about some more recent work.

8th, February, 2022(Tuesday)

Jihun Yum (IBS, Center for Complex Geometry)

Limits of Bergman kernels on a tower of coverings of compact Kähler manifolds

The Bergman kernel B_X, which is by the definition the reproducing kernel of the space of L^2 holomorphic n-forms on a n-dimensional complex manifold X, is one of the important objects in complex geometry. In this talk, we observe the asymptotics of the Bergman kernels, as well as the Bergman metric, on a tower of coverings. More precisely, we show that, for a tower of finite Galois coverings $\{\phi_j : X_j \to X\}$ of compact Kähler manifold X converging to an infinite Galois covering $\phi : \widetilde{X} \to X$, the sequence of push-forward Bergman kernels $\phi_{j*} B_{X_j}$ locally uniformly converges to $\phi_* B_{\widetilde{X}}$. Also, we show that if the canonical line bundle $K_{\widetilde{X}}$ of $\widetilde{X}$ is very ample, then the canonical line bundle K_{X_j} of X_j is also very ample for sufficiently large j. This is a joint work with S. Yoo in IBS-CCG.

Hoseob Seo (Seoul National University)

On L^2 extension from singular hypersurfaces

In L^2 extension theorems from an irreducible singular hypersurface in a complex manifold, important roles are played by certain measures such as the Ohsawa measure, which determines when a given function can be extended. In this talk, we show that the singularity of the Ohsawa measure can be identified in terms of algebraic geometry. Using this, we give an analytic proof of the inversion of adjunction in this setting. These considerations enable us to compare various positive and negative results on L^2 extension from singular hypersurfaces. In particular, we generalize a recent negative result of Guan and Li which places limitations on strengthening such L^2 extension by employing a less singular measure in the place of the Ohsawa measure. This is joint work with Dano Kim.

Félix Lequen (CY Cergy Paris Université)

Almost every K3 surface contains infinitely many linear Levi-flat hypersurfaces

In this talk, I will briefly explain a beautiful recent construction of K3 surfaces by Koike and Uehara, obtained by gluing in a manner reminiscent of surgery in topology, which is made possible by Arnol'd linearisation theorem under a certain Diophantine condition. I will permits, I will explain how to use ergodic theory following Verbitsky to show that in a certain sense, almost every K3 surface contains infinitely many linear Levi-flat hypersurfaces, a certain interesting class of hypersurfaces in complex manifolds.

Olivier Thom (Instituto de Matemática Pura e Aplicada)

Neighborhoods of higher genus curves: formal foliations and classification

We want to classify 2-dimensional neighborhoods of smooth compact complex curves of genus at least 2 with trivial normal bundle. The strategy is first to construct smooth foliations tangent to the curve, fix two of them (choosen as canonically as possible) and then show that the moduli space of neighborhoods equipped with two such foliations can be explicitely computed, using holonomy-like invariants.

I will explain how we can always construct formal foliations in this context, why we can choose some of them canonically, and how this gives a formal classification of such neighborhoods.

9th, February, 2022(Wednesday)

Seungjae Lee (IBS, Center for Complex Geometry)

Symmetric differentials on complex hyperbolic space forms and its L^2 holomorphic jet extension

Consider a compact complex ball quotient Σ under the action of a cocompact and torsion-free lattice $\Gamma \subset \mathrm{Aut}(\mathbb{B}^n)$. Since the complex unit ball has a Kähler metric induced by the Bergman metric on $\mathbb{B}^n$, it becomes a complex hyperbolic space form. Moreover, Σ has a ball fiber bundle Ω by the diagonal action of Γ for $\mathbb{B}^n \times \mathbb{B}^n$. Also, Ω has the maximal compact analytic subvariety D which is biholomorphic to Σ.

When Σ is a compact Riemann surface, M. Adachi (2017) proved that a given holomorphic jet on D induces a weighted L^2 holomorphic function on Ω using some recursive formula for $\bar{\partial}$-operator. Applying this result, he showed that Ω has no any non-constant bounded holomorphic function.

In this talk, I will extend these results for n-dimensional compact complex hyperbolic forms by using a Hodge type identity $\mathcal{R}_G$ for symmetric powers of T_Σ^* and a suitable recursive formula which is compatible with $\mathcal{R}_G$. Moreover, as another application of these schemes, I will show the existence of symmetric differentials of degree $m \geq n+1$ on Σ under certain conditions using a Poincaré series on Ω. This is joint work with Aeryeong Seo of Kyungpook National University.

Lukas Braun (University of Freiburg)

Kähler metrics, orbifold structures, and fundamental groups on resolutions of klt spaces

I will discuss how for complex algebraic varieties with singularities of klt type, orbifold structures on log resolutions provide a natural framework regarding questions about fundamental groups and Kähler metrics, without considering the initial singular variety at all.

10th, February, 2022(Thursday)

Genki Hosono (Tohoku University)
On the characterization of Griffiths positivity of Hermitian metrics

Recent studies show that curvature positivity conditions and L^2 theoretic conditions for Hermitian holomorphic vector bundles are related in some interesting ways. In this talk, I will explain these conditions and their relationship. Our main result states that the Griffiths semi-positivity is characterized by some asymptotic L^2 theoretic conditions, including the symmetric multiple coarse L^2-estimate (and L^2-extension) condition. This talk is based on a joint study with Takahiro Inayama.

Zhiwei Wang (Beijing Normal University)
Recent progress on extension of quasiplurisubharmonic functions on Kähler manifolds

In this talk, we will present some recent progress on the extension of quasiplurisubharmonic functions on Kähler manifolds and related problems. These are base on joint work with Prof. Fusheng Deng, Jiafu Ning, Xiankui Meng and Xiangyu Zhou.

Martin Sera (Kyoto University of Advanced Science)
A smoothness criterion for complex spaces in terms of differential forms

On a reduced complex space X, weakly holomorphic 1-forms are holomorphic forms on the regular part X_{reg} that extends holomorphically across the exceptional set of a resolution of singularities of X. In a joint work with H. Samuelsson Kalm, we show that if the sheaf of germs of weakly holomorphic 1-forms on X is locally free (or other certain sheaves of holomorphic 1-forms), then X is smooth. We will relate this observation with the Zariski-Lipman conjecture which claims a complex space with locally free tangent sheaf is smooth.

Juanyong Wang (Chinese Academy of Sciences)
On the structure of varieties of semi-Fano type

A normal projective variety X is called of semi-Fano type, if there is an effective $\mathbb{Q}$-divisor D on X such that (X, D) is klt and $-(K_X + D)$ is nef. The motivation of studying these varieties is two-folded: on one hand, to extend the singular Beauville-Bogomolov decomposition to the log case, on the other hand, to comprehend the structure of varieties with semi-positive curvature. If X is smooth, the structure of these varieties is obtained by the successive works of Demailly-Peternell-Schneider, Pǎun, Debarre, Zhang, Cao, Cao-Höring, Campana-Cao-Matsumura, etc.. In fact, the Albanese map and the MRC fibration of these varieties are shown to be locally trivial fibrations, and induce a decomposition of their universal covers. By the philosophy of MMP, it is then natural to extend these results to the singular case, that is, the case where X is of semi-Fano type. In a recent work with Shin-ichi Matsumura, we obtain a structure theorem for these varieties by proving that the MRC fibration is a locally trivial fibration; moreover, by combining this with the results of Ambro (2005), we obtain a splitting theorem for varieties of log Calabi-Yau type. In this talk, I will explain these results and present some applications to related problems.