Workshop on Dynamics in Arithmetic and Complex Geometry and its applications

16th–19th January 2024, @E408 Osaka Metropolitan Univ.[1]

Program

January 16th (Tuesday)

13:30–13:55 Registration

14:00–15:00 Keita Goto (National Taiwan University)
Non-Archimedean Analog of SYZ Fibration

15:15–16:15 Eiji Inoue (RIKEN iTHEMS)
Recent developments of Kähler-Ricci flow on Fano manifolds

16:30–17:30 Jinichiro Tanaka (Osaka Metropolitan University)
Cohomology groups with compact support for flat line bundles on certain complex lie groups

January 17th (Wednesday)

10:00–11:00 Yûsuke Okuyama (Kyoto Institute of Technology)
Equilibrium measures for reduced complex dynamics

11:10–12:10 Shunsuke Usuki (Kyoto University)
On a lower bound of the number of integers in Littlewood's conjecture

14:00–15:00 Reimi Irokawa (NTT institute for fundamental mathematics)
Hybrid dynamics of hyperbolic automorphisms of K3 surfaces

15:15–16:15 Chih-Chiang Kao (Tokyo Institute of Technology)
Iterated Galois groups of $X^2 + c$ over quadratic number fields

16:30–17:30 Yu Yasufuku (Nihon University)
Uniform boundedness of integral points in orbits

Banquet

January 18th (Thursday)

10:00–11:00 Yuta Takada (Hokkaido University)
Characteristic polynomials of isometries of lattices and dynamical degrees of automorphisms of K3 surfaces

11:10–12:10 Takato Uehara (Okayama University)
Dynamical spectrum on projective surfaces

14:00–15:00 Takahiro Shibata (National Fisheries University)
The classification of smooth quotients of abelian surfaces

15:30–16:30 Tatsuro Kawakami (Kyoto University)
Endomorphisms of varieties and Bott vanishing

January 19th (Friday)

10:00–11:00 Yuta Kusakabe (Kyoto University)
Dynamical methods in Gromov's elliptic geometry

11:10–12:10 Rin Gotou (Osaka University)
Nonlinear Algebra on Multiplier Map on Cubic Endomorphisms on the Projective Line

Abstracts

Keita Goto (National Taiwan University)
Non-Archimedean Analog of SYZ Fibration

SYZ fibrations have been studied for long years in a context of so-called SYZ mirror symmetry. In particular, this framework of mirror symmetry is expected to work for degenerating families of polarized Calabi–Yau varieties. Non-Archimedean geometry offers an analytic way to treat them as if they were just varieties over a discrete valued field. In this talk, we define a non-Archimedean analog of such an SYZ fibration and compare these two kinds of SYZ fibrations via Berkovich Geometry in a broader sense, which we call Hybrid Geometry.

Rin Gotou (Osaka University)
Nonlinear Algebra on Multiplier Map on Cubic Endomorphisms on the Projective Line

A generic cubic endomorphism on the projective line is distinguished from other endomorphisms by the set of its multipliers. We see this in an aspect of nonlinear algebra, application of classical and computational invariant theory to moduli spaces of dynamical systems.

Eiji Inoue (RIKEN iTHEMS)
Recent developments of Kähler-Ricci flow on Fano manifolds

I will review recent developments on the analysis of the time infinite limit behavior of (normalized) Kähler–Ricci flow on Fano manifolds, which I believe is one of the most brilliant discoveries in the history of complex geometry. It involves various fields: geometric analysis, metric geometry, pluripotential theory, non-archimedean pluripotential theory and MMP.

Reimi Irokawa (NTT institute for fundamental mathematics)
Hybrid dynamics of hyperbolic automorphisms of K3 surfaces

In this talk, we treat a family of complex K3 surfaces with hyperbolic automorphisms parametrized by the open unit disk possibly meromorphically degenerating at the origin. We use the theory of hybrid spaces, invented by Boucksom, Favre and Jonsson to study the limiting behavior of such families of dynamical systems, especially the "weak limit" of the invariant measures.

Chih-Chiang Kao (Tokyo Institute of Technology)
Iterated Galois groups of $X^2 + c$ over quadratic number fields

Let $K = \mathbb{Q}(\sqrt{d})$ be quadratic number fields, and $f(X) = X^2 + c$ be polynomials with $c \in \mathbb{Z}[\sqrt{d}]$. Denote $f^n(X)$ as the n-th iterate of $f(X)$. Building upon the work of Odoni and Stoll, we provide criteria concerning the constant terms of $f^n(X)$ to determine whether the Galois group $\mathrm{Gal}(f^n(X)/K)$ is isomorphic to $[C_2]^n$, the n-th fold wreath product of cyclic groups.
We then apply this result to quadratic number fields K with odd class numbers except for the case when $d = -1$, and demonstrate that $\mathrm{Gal}(f^n(X)/K) \cong [C_2]^n$ provided c satisfies certain conditions. Finally, we prove that for any quadratic number fields K, there exists a certain proportion of integers c for which $\mathrm{Gal}(f^n(X)/K) \cong [C_2]^n$ for all $n \in \mathbb{Z}$.

Tatsuro Kawakami (Kyoto University)
Endomorphisms of varieties and Bott vanishing

In this talk, we show that a projective variety with an int-amplified endomorphism of degree invertible in the base field satisfies Bott vanishing. This is a new way to analyze which varieties have nontrivial endomorphisms. In particular, we extend some classification results on varieties admitting endomorphisms (for Fano threefolds of Picard number one and several other cases) to any characteristic. This talk is based on a joint work with Burt Totaro.

Yuta Kusakabe (Kyoto University)
Dynamical methods in Gromov's elliptic geometry

As an opposite property of Kobayashi–Eisenman–Brody hyperbolicity, Gromov introduced the notion of ellipticity for complex manifolds and algebraic varieties in 1989. Briefly, Gromov's ellipticity means the existence of many "sprays" which generalize $\mathbb{G}_a$-actions. In this talk, we will first review Gromov's elliptic geometry in both analytic and algebraic settings, and then discuss applications of dynamical methods in this geometry.

Yûsuke Okuyama (Kyoto Institute of Technology)
Equilibrium measures for reduced complex dynamics

We determine all the equilibrium measures for a reduced complex dynamical system, which is the reduction of a holomorphic family of rational functions meromorphically parametrized around a puncture in $\mathbb{C}$. We also give a proof of the weak convergence of their equilibrium measures in $\mathbb{P}^1$, which is asserted by DeMarco and Faber.

Takahiro Shibata (National Fisheries University)
The classification of smooth quotients of abelian surfaces

Quotients of abelian varieties by finite groups are interesting in dynamical point of view since they have polarized endomorphisms. In this talk, we see the classification of the smooth quotients of abelian surfaces, focusing on their augmented irregularities.

Yuta Takada (Hokkaido University)
Characteristic polynomials of isometries of lattices and dynamical degrees of automorphisms of K3 surfaces

In connection with the study of automorphisms of complex surfaces, the question of which polynomial can be realized as the characteristic polynomial of an even unimodular lattice has been considered. Recently, by using the idea of local-global principle, E. Bayer–Fluckiger gave an answer to this question when the constant term of the polynomial is 1. In this talk, we extend her result to the case where the constant term is not 1, i.e., -1. We will also discuss its application to dynamical degrees of automorphisms of K3 surfaces.

Jinichiro Tanaka (Osaka Metropolitan University)
Cohomology groups with compact support for flat line bundles on certain complex lie groups

Let X be a complex surface obtained as the quotient of the complex Euclidean space $\mathbb{C}^2$ by a discrete subgroup Γ of rank 3. We investigate the cohomology group $H_0^1(X, E)$ with compact support for a unitary flat line bundle E over X. We show the vanishing of $H_0^1(X, E)$ for a certain class of such pairs (X, E) under irrational number theoretical conditions on Γ. The class of (X, E) as above includes infinitely many examples such that $H^1(X, E)$ is non-Hausdorff and infinite dimensional.

Takato Uehara (Okayama University)
Dynamical spectrum on projective surfaces

For a birational mapping on a projective surface, one can define its dynamical degree, which measures the complexity of dynamical behavior of the mapping. The set of dynamical degrees of birational mappings, called the dynamical spectrum, has properties similar to that of volumes of hyperbolic 3-manifolds, shown by Thurston, and also that of growth rates of finite generators of a non-elementary hyperbolic group, shown by Fujiwara–Sela. In this talk, we will explain these properties of the dynamical spectrum.

Shunsuke Usuki (Kyoto University)
On a lower bound of the number of integers in Littlewood's conjecture

Littlewood's conjecture is a famous and long-standing open problem in simultaneous Diophantine approximation. Actually, this problem is closely related to the action of diagonal matrices on $\mathrm{SL}(3, \mathbb{R})/\mathrm{SL}(3, \mathbb{Z})$, and a groundbreaking result toward this problem was shown by Einsiedler, Katok and Lindenstrauss from the "measure rigidity" for this action. In this talk, I will explain that a new quantitative result on Littlewood's conjecture can be obtained from the diagonal action. The keys are the measure rigidity and further studies on behavior of empirical measures for this action.

Yu Yasufuku (Nihon University)
Uniform boundedness of integral points in orbits

For a morphism ϕ on $\mathbb{P}^N$ defined over a number field, we discuss several uniformity results for integral points in orbits, generalizing Hsia–Silverman's result for $N = 1$. Here, "uniformity" is in two senses: first, the bound is independent of the initial point of the orbit, and second, it is uniform across all number fields of bounded degree. For the proofs, we employ Le's Wirsing - type theorem, Matsuzawa's use of asymptotic multiplicity, and Schmidt subspace theorem in the Ru–Vojta framework.