Problems on foliations and dynamics in complex geometry

20th–22nd November 2023, @Room 111, RIMS[1]

Program
※ Each talk is 50 min (+ 10 min discussion).

November 20th (Monday)

9:00 − 12:00: Free Discussion

13:00 − 13:30: Registration

13:30 − 14:30: Takayuki Koike (Osaka Metropolitan University)
$\bar{\partial}$ cohomology of the complement of a semi-positive anticanonical divisor of a compact surface

14:50 − 15:50: Satoshi Ogawa (Osaka Metropolitan University)
On tubular neighborhoods of compact complex curves and Brjuno condition

16:00 − 17:00: Tetsuo Ueda (Kyoto University)
Neighborhood of a compact curve with singularities

November 21st (Tuesday)

9:00 − 10:00: Jinichiro Tanaka (Osaka Metropolitan University)
Dual spaces of cohomology groups for flat line bundles on certain complex Lie groups

10:10 − 11:10: Shota Kikuchi (Suzuka National College of Technology)
On the Azukawa pseudometric defined from the pluricomplex Green function with poles along subvarieties

11:20 − 12:20: Kazuhisa Miyazawa
A remark on the paper "On the infinite dimensionality of the middle L^2 cohomology of complex domains"

13:30 − 14:30: Yuta Kusakabe (Kyoto University)
Gromov's elliptic geometry from the viewpoints of foliations and dynamics

14:50 − 15:50: Seungjae Lee (IBS Center for Complex Geometry)
A version of L^2-Hodge theory for complex hyperbolic space forms with finite volume

16:00 − 17:00: Kengo Hirachi (The University of Tokyo)
Normal scale for pseudo-Einstein contact forms and intrinsic CR normal coordinates

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November 22nd (Wednesday)

9:00 − 10:00: Daisuke Tarama (Ritsumeikan University)
Complex geometry around rigid body dynamics

10:10 − 11:10: Hiromichi Mizuno (Kyushu University)
Newton polyhedra and oscillatory integrals with rational phases

11:20 − 12:20: Joe Kamimoto (Kyushu university)
A new boundary invariant and the growth of the Bergman kernel

13:30 − 17:00: Free Discussion

Abstracts

Kengo Hirachi (The University of Tokyo)
Normal scale for pseudo-Einstein contact forms and intrinsic CR normal coordinates

In conformal geometry, Robin Graham gave a choice of good scale in the conformal class to simplifies the computation involving the jets of the conformal structure. We will give an analogeous result in CR geometry based on an earlier work by Jerison–Lee. Our choice of the contact form is simpler as we can stay in the class of pseud-Einstein contact forms, which was not the case for the choice of Jerison–Lee. We also plan to explain its application to the Szegö kernel.

Joe Kamimoto (Kyushu university)
A new boundary invariant and the growth of the Bergman kernel

In the study of several complex variables, it is an important issue to describe analytic invariants in terms of geometric invariants. In this talk, in order to describe the growth order of the Bergman kernel at the boundary, we introduce some geometric invariant of the boundary. Moreover, we show that the above growth order can be described in terms of our invariant in many cases.

Shota Kikuchi (Suzuka National College of Technology)
On the Azukawa pseudometric defined from the pluricomplex Green function with poles along subvarieties

The Azukawa pseudometric is a function defined from the pluricomplex Green function with a pole at a point, and it is a generalization of the Robin constant defined from the classical Green function. The Azukawa pseudometric is useful to analyze behavior of the pluricomplex Green function near its a pole, and it has deep connections with important objects in complex analysis such as the Carathéodory pseudometric, the Kobayashi pseudometric and the Bergman kernel. In this talk, we consider the counterpart of the Azukawa pseudometric for the pluricomplex Green function with poles along subvarieties, and explain about its properties and applications.

Takayuki Koike (Osaka Metropolitan University)
$\overline{\partial}$ cohomology of the complement of a semi-positive anticanonical divisor of a compact surface

Let X be a non-singular compact complex surface such that the anticanonical line bundle admits a smooth Hermitian metric with semi-positive curvature. For a non-singular hypersurface Y which defines an anticanonical divisor, we investigate the $\overline{\partial}$ cohomology group $H^1(M, \mathcal{O}_M)$ of the complement $M = X \setminus Y$.

Yuta Kusakabe (Kyoto University)
Gromov's elliptic geometry from the viewpoints of foliations and dynamics

As an opposite property of Kobayashi–Eisenman–Brody hyperbolicity, Gromov introduced the notion of ellipticity for complex manifolds in 1989. Briefly, Gromov's ellipticity means the existence of many "holomorphic sprays" which generalize holomorphic $\mathbb{C}$-actions. In this talk, we will first review Gromov's elliptic geometry from this viewpoint of dynamics, and then discuss the generalization of Gromov's ellipticity to holomorphic foliations.

Seungjae Lee (IBS Center for Complex Geometry)

A version of L^2-Hodge theory for complex hyperbolic space forms with finite volume

In this talk, we discuss a version of L^2-Hodge theory for compact hyperbolic space forms with finite volume. Hodge theory for compact complex manifolds gives the canonical isomorphism between Dolbeault cohomologies and the spaces of harmonic forms. This theory has been extended to L^2-Dolbeault cohomologies for a non-compact Kähler manifold that possesses a global Kähler potential with bounded gradient for the metric, by several scholars.

To extend these results for L^2-Dolbeault cohomologies associated with the symmetric power of the cotangent bundle of complex hyperbolic space forms with finite volume, we establish a cohomological isomorphism theorem between a L^2-Dolbeault cohomology and a sheaf cohomology of a certain holomorphic vector bundle for its toroidal compactification.

Moreover, we extend previous results concerning the dimensionality, both infinite and finite, of weighted Bergman spaces on compact complex hyperbolic space forms to non-compact hyperbolic spaces forms with finite volume. This is a joint work with Aeryeong Seo of Kyungpook National University.

Kazuhisa Miyazawa

A remark on the paper "On the infinite dimensionality of the middle L^2 cohomology of complex domains"

We will give a detailed explanation of the argument of Ohsawa's paper by using a family of smooth boundaries and Lefschetz operator-like actions. Then we will find some properties and examples of the infinite dimensional reduced and unreduced L^2 cohomology.

Hiromichi Mizuno (Kyushu University)

Newton polyhedra and oscillatory integrals with rational phases

In this talk, we will investigate the asymptotic behavior of oscillatory integrals by using the resolution of singularities based on the geometry of Newton Polyhedra. We will explain the history of analysis of oscillatory integrals and introduce the work of Varchenko which is the basis of our analysis. After that, we try to generalize his work in the case when the phase function is rational functions. This is a joint work with Joe Kamimoto.

Satoshi Ogawa (Osaka Metropolitan University)

On tubular neighborhoods of compact complex curves and Brjuno condition

Linearization of horizontal and transverse transition functions along a neighborhood of a compact complex manifold is called full-linearization. We can get a holomorphic tubular neighborhoods of compact complex curve by full-linearization. Gong–Stolovitch gave a sufficient condition for full-linearization by focusing on estimates of operator norms of Čech coboundary map with L^2 norm. In this talk, I will talk about the Brjuno condition of unitary flat line bundles from the viewpoint of full-linearization along a compact complex curve.

Jinichiro Tanaka (Osaka Metropolitan University)

Dual spaces of cohomology groups for flat line bundles on certain complex Lie groups

Let X be a complex surface obtained as the quotient of the complex Euclidean space $\mathbb{C}^2$ by a discrete subgroup of rank 3. We investigate the topological dual $(H^1(X, E))^\vee$ of the cohomology group $H^1(X, E)$ for a unitary flat line bundle E over X. We show the vanishing of $(H^1(X, E))^\vee$ for a certain class of such pairs (X, E), which includes infinitely many examples of (X, E) such that $H^1(X, E)$ is non-Hausdorff and infinite dimensional.

Daisuke Tarama (Ritsumeikan University)

Complex geometry around rigid body dynamics

This talk gives an overview about the complex algebro-geometric aspects of completely integrable systems describing the rotational motion of a rigid body, based on the speaker's works with Isao Naruki and Jean–Pierre Françoise. Particularly in the case of the Euler rigid body, the geometry of the associated elliptic fibrations is investigated. If time permits, further developments will also be mentioned.

Tetsuo Ueda (Kyoto University)

Neighborhood of a compact curve with singularities

Let C be a connected compact complex curve in a complex manifold of dimension 2. We will show that, if the intersection matrix of the curve C has a positive eigenvalue, then there is a neighborhood V of C and a strictly plurisubharmonic function on $V \setminus C$ that increases logarithmically near C. (Joint work with Takayuki Koike)