

# Non-archimedean and tropical geometry from complex and algebraic geometry and dynamics<sup>1</sup>

20th–24th January 2025, @E408 Osaka Metropolitan Univ.

MONDAY, JANUARY 20

14:00–15:00 上原崇人 **Takato Uehara**

*Siegel domains around invariant curves*

15:30–16:30 小池貴之 **Takayuki Koike**

*Formal principle for line bundles on neighborhoods of an analytic subset of a compact Kähler manifold*

TUESDAY, JANUARY 21

10:00–11:00 奥山裕介 **Yûsuke Okuyama**

*Meromorphic degeneration of non-archimedean dynamics*

11:15–12:15 三原朋樹 **Tomoki Mihara**

可換  $C^*$  環の導来テイト非輪状性 (*Derived Tate acyclicity for commutative  $C^*$ -algebras*)

14:00–15:00 宍倉光広 **Mitsuhiro Shishikura** (special talk)

*Rescaling limits of quadratic rational maps and trees of spheres*

15:30–16:30 橋本義規 **Yoshinori Hashimoto**

*Coupled Kähler-Einstein metrics and stability*

WEDNESDAY, JANUARY 22

10:00–11:00 井上瑛二 **Eiji Inoue**

*Chromatic  $W$  and  $\mu$ -entropy*

11:15–12:15 久本智之 **Tomoyuki Hisamoto**

*T. B. A.*

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<sup>1</sup>This conference is supported by Osaka Central Advanced Mathematical Institute (MEXT Joint Usage/Research Center on Mathematics and Theoretical Physics).

14:00–15:00 尾高悠志 **Yuji Odaka** (special talk)  
*カラビヤウ多様体・計量の極限 への非アルキメデス・トロピカル幾何学の応用 (Applications of non-archimedean / tropical ideas to asymptotics of Calabi–Yau metrics)*

15:30–16:30 後藤慶太 **Keita Goto**  
*An application of non-Archimedean geometry to the metric SYZ conjecture for certain toric degenerations*

THURSDAY, JANUARY 23

10:00–11:00 中島康仁 **Yasuhito Nakajima**  
*Krull dimension in tropical geometry*

11:15–12:15 宋珠愛 **JuAe Song**  
*An algebraic characterization of rational function semifields of tropical curves*

14:00–15:00 山木吉彦 **Kazuhiko Yamaki** (special talk)  
*Effective faithful tropicalizations and embeddings for abelian varieties*

15:30–16:30 三上陵太 **Ryota Mikami**  
*Numerical equivalence and tropical intersection cohomology*

FRIDAY, JANUARY 24

10:00–11:00 松澤陽介 **Yohsuke Matsuzawa**  
*Dynamical Lang-Siegel type theorem for rational maps on projective varieties*

11:15–12:15 ヘルミンクポール **Paul Helminck**  
*Finding pair-of-pants decompositions of algebraic varieties using tropical geometry*

## Abstracts

上原崇人 **Takato Uehara**

*Siegel domains around invariant curves*

In this talk, we will deal with automorphisms on rational surfaces admitting rotation domains, which are Fatou components on which the automorphisms act by irrational rotations. The aim of this talk is to discuss whether rotation domains appear around invariant curves under the assumption that these automorphisms have positive entropy.

小池貴之 **Takayuki Koike**

*Formal principle for line bundles on neighborhoods of an analytic subset of a compact Kähler manifold*

We investigate the formal principle for holomorphic line bundles on neighborhoods of an analytic subset of a complex manifold mainly when it can be realized as an open subset of a compact Kähler manifold.

奥山裕介 **Yūsuke Okuyama**

*Meromorphic degeneration of non-archimedean dynamics*

In arithmetic study of the space or the moduli of dynamics of rational functions, it is important to study their degeneration in both complex and non-archimedean setting. Especially, the at most logarithmic divergence and the (dis)continuity property of the Lyapunov exponent functions for meromorphic degeneration have been intensively studied. In this talk, we talk about some details on this topic in non-archimedean setting.

三原朋樹 **Tomoki Mihara**

*可換  $C^*$  環の導来テイト非輪状性 (Derived Tate acyclicity for commutative  $C^*$ -algebras)*

従来の非アルキメデス幾何において、空間の貼り合わせは層性と呼ばれる完備テンソルに関する性質を用いて定式化される。特に完備付値体上のリジッド幾何においてはアフィノイド代数と呼ばれる良い有限的な代数は層性より強くテイト非輪状性を満たすことが知られており、これにより空間の貼り合わせを自由に行うことが可能である。一方で無限的な代数は必ずしも層性を満たさないことが知られており、空間を自由に貼り合わせられないという問題がある。この問題を解決する方法はいくつかあり、例えばパーフェクトイド代数に限ればテイト非輪状性を満たすことを P.Scholze 氏が示し、より一般に安定一様代数に限ればテイト非輪状性を満たすことを私（および独立に K.Buzzard 氏と A.Verberkmoes 氏）が示した。本講演では更に P.Scholze 氏, F.Bambozzi 氏, O.Ben-Bassat 氏, K.Kremnizer 氏らによって近年研究されている別のアプローチとして、テイト非輪状性の定義に現れる「同型」を適切に「ホモトピー同値」に置き換えることで得られる導来テイト非輪状性を用いた空間の貼り合わせの新たな定式化を紹介し、可換  $C^*$  環が導来テイト非輪状性を満たすことを説明し、応用として可換  $C^*$  環上の加群圏におけるディセント問題を扱う。本研究は F.Bambozzi 氏との共同研究である。

In standard formulation of non-Archimedean geometry, we glue spaces using sheafiness, which is a property on completed tensor products. Especially in rigid geometry over a complete valuation field, we use as the building blocks affinoid algebras, which are topologically finitely generated algebras and are classically known to satisfy an important property called Tate acyclicity. Tate acyclicity implies sheafiness, and hence allows us to glue spaces freely. On the other hand, when we deal with general algebras which are possibly topologically infinitely generated, the sheafiness does not necessarily hold, and we need to face the problem that we cannot glue spaces freely. There are several ways to solve this problem. For example, P. Scholze proved that perfectoid algebras satisfy Tate acyclicity, and more generally, I (and also independently K. Buzzard and A. Verberkmoes) proved that stably uniform algebras satisfy Tate acyclicity. In my talk, I will introduce another solution recently studied by P. Scholze, F. Bambozzi, O. Ben-Bassat, K. Kremnizer, and so on, based on a new formulation of gluing spaces using derived Tate acyclicity, a derived variant of Tate acyclicity, defined by replacing “isomorphism” in the definition of Tate acyclicity by “homotopy equivalence” in some sense. Further, I will explain that commutative  $C^*$ -algebras satisfy derived Tate acyclicity, and apply the result to a descent problem in the module categories over commutative  $C^*$ -algebras. This is a joint work with F. Bambozzi.

宍倉光広 **Mitsuhiro Shishikura** (special talk)

*Rescaling limits of quadratic rational maps and trees of spheres*

When a family of rational maps degenerates, certain parametrized coordinate changes may give rise to a non-trivial return map. J. Kiwi studied such scaling limits for quadratic rational maps and M. Arfeux defined “trees of spheres” for the degeneration. We will discuss a converse problem which means a construction of degeneration family from a given data, and its relation to the Berkovich space of the extension of the field of Laurent series. This is a work in progress with Arfeux and Kiwi, and some work-out examples with E. Hironaka related to  $\text{Per}_n(0)$ .

橋本義規 **Yoshinori Hashimoto**

*Coupled Kähler-Einstein metrics and stability*

A foundational theorem in Kähler geometry states that a Kähler-Einstein metric exists on a Fano manifold (with discrete automorphisms) if and only if it is uniformly Ding stable, which happens if and only if the stability threshold is larger than 1. In the absence of Kähler-Einstein metrics, we can seek coupled Kähler-Einstein metrics defined in terms of a decomposition of the anticanonical bundle. The main result of this talk is the equivalence between the coupled Ding stability, the coupled stability threshold larger than 1 (once they are appropriately defined), and the existence of coupled Kähler-Einstein metrics. The proof relies on the theory of non-Archimedean metrics, as in the original case of Kähler-Einstein metrics. This is a joint work with Kento Fujita.

井上瑛二 **Eiji Inoue**

*Chromatic  $W$  and  $\mu$ -entropy*

I introduce chromatic  $W$ -entropy and explain a non-archimedean pluripotential theoretic

aspect of this functional. This is a generalization of previous research on Perelman's entropy in Kähler geometry.

久本智之 **Tomoyuki Hisamoto**

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尾高悠志 **Yuji Odaka** (special talk)

カラビヤウ多様体・計量の極限 への非アルキメデス・トロピカル幾何学への応用 (*Applications of non-archimedean / tropical ideas to asymptotics of Calabi–Yau metrics*)

アーベル多様体や K3 曲面に代表される広義でカラビヤウな射影代数多様体の (退化) 族が与えられた時, その良い極限 (中心ファイバー) を作り出すということは, 代数幾何では古典的な問題 (MMP などが使える) であるが, その構成に標準性・一意性を見出すのは一般には極めて困難である. しかし微分幾何で古典的なカラビヤウ計量という標準的な計量の極限を取ると, いわば恣意的な選択をすることなく標準的な極限「空間」が自然に取り出せるはずである. そして興味深いことにこうした空間構造はしばしばまだ代数的な構造を持ち, 代数多様体論に加え, 非アルキメデス幾何学, トロピカル幾何学的なアイデアを持って初めて (より) よく理解される. こうした現象は, ケーラー幾何における K 安定性理論の発展の延長にあると同時に, ミラー対称性に対する Kontsevich–Soibelman らの洞察の延長にあることでもあり, その融合の可能性を感じさせる.

この講演では特に, 二つの異なるタイプの退化族のクラスに対して, より非アルキメデス幾何・トロピカル幾何的な発想を取り入れることで, カラビヤウ計量の極限を記述 (ないし予想を作り出す) するという二つの例 (arXiv:2010.00416, arXiv:2406.14518) を紹介したい. 前者は K3 曲面の type II 退化の場合であり, 大島芳樹氏との共同研究に深く関わる. 特にあるクラスの 1 変数 24 次多項式のトロピカル化の研究が, 計量の複雑な漸近挙動を決定する. 後者は正規 (KLT)・既約なままの退化を考察する. 微分幾何的には崩壊やバブルと呼ばれる状況だが, それらの予備知識はあまり仮定しない予定である. 後藤慶太氏の講演にもつながる.

後藤慶太 **Keita Goto**

*An application of non-Archimedean geometry to the metric SYZ conjecture for certain toric degenerations*

Berkovich geometry provides an analytic framework for addressing problems in algebraic geometry. As such an example, we consider what is called the metric SYZ conjecture. This is a metric version of Strominger–Yau–Zaslow conjecture, which is a central problem in mirror symmetry. Yang Li proposed this metric SYZ conjecture and established its sufficient condition using non-Archimedean geometry. In this talk, I will explain our result on the sufficient condition for toric degenerations of Calabi–Yau complete intersections. This talk is based on joint work with Yuto Yamamoto.

中島康仁 **Yasuhito Nakajima**

*Krull dimension in tropical geometry*

As in classical algebraic geometry, congruences on the tropical Laurent polynomial semiring define congruence varieties in the tropical algebraic torus. If a congruence on the tropical Laurent polynomial semiring has a finite tropical basis, the corresponding congruence variety naturally has its dimension because it forms the support of a polyhedral complex. In this talk, we establish the relationship between the Krull dimension of the quotient semiring by a congruence having a finite tropical basis and the dimension of the corresponding congruence variety. This is joint work with JuAe Song.

**宋珠愛 JuAe Song**

*An algebraic characterization of rational function semifields of tropical curves*

In the classical algebraic geometry, it is well-known that function fields of algebraic curves are characterized with transcendental degree. On the other hand, since there is no appropriate analogue of transcendental degree for semifield extensions, we cannot characterize rational function semifields of (abstract) tropical curves as in the classical case. In this talk, we give an algebraic characterization of rational function semifields of tropical curves avoiding the concept of transcendental degree and explain mainly the conditions corresponding to the connectivity of tropical curves and to the condition without parallel rays.

**山木 孝彦 Kazuhiko Yamaki** (special talk)

*Effective faithful tropicalizations and embeddings for abelian varieties*

In a recent joint work with Shu Kawaguchi, we give an answer to a faithful tropicalization problem for the canonical skeleton of an abelian variety over a complete nonarchimedean field with nontrivial absolute value. We construct a faithful tropicalization by using nonarchimedean theta functions, and our result can be seen as a tropical version of the classical theorem of Lefschetz for complex abelian varieties. The key ingredients of the proof are faithful embeddings of tropical abelian varieties by tropical theta functions and lifting of tropical theta functions to nonarchimedean theta functions. In this talk, I will overview the faithful tropicalization problem, focusing on the those results for abelian varieties.

**三上陵太 Ryota Mikami**

*Numerical equivalence and tropical intersection cohomology*

In this talk, I would like to discuss geometric descriptions of algebraic cycles with rational coefficients modulo numerical equivalence, which is defined formally by intersection numbers, for smooth projective varieties. In the classical toric case, they are isomorphic to singular cohomology. We generalize it to suitable pairs of smooth projective varieties and simple normal crossing divisors using tropical analog of intersection cohomology.

**松澤陽介 Yohsuke Matsuzawa**

*Dynamical Lang-Siegel type theorem for rational maps on projective varieties*

Dynamical Lang-Siegel theorem is a theorem about the growth rate local height function (i.e.  $-\log$  of  $v$ -adic distance from given subvariety) along orbits of self-maps. For orbits of rational points, it cannot grow as fast as Weil height function, the arithmetic complexity of rational

points. We will talk about such a theorem for rational maps on projective varieties.

ヘルミンクポール **Paul Helminck**

*Finding pair-of-pants decompositions of algebraic varieties using tropical geometry*

In this talk, I will show how one can use tropical geometry to construct an explicit CW-complex that realizes the homotopy type of an algebraic variety. This technique conjecturally works for all algebraic varieties, and it generalizes the well-known pair-of-pants decomposition for compact Riemann surfaces. The main ingredients in the construction are:

- Degenerations of hyperplane arrangements and complex matroids,
- Kato-Nakayama spaces, and
- Smooth tropicalizations.

I will show how all of these enter the picture, and how they work in practice for various varieties, including K3-surfaces. In this talk, I will focus on the local aspects of the construction, but if time permits I will also briefly discuss how one can glue these local parts using the new notion of a universal complex matroid that is independent of the chosen local coordinate system. This is part of an ongoing project with Yassine El Maazouz.