Your talk title

Definitions, theorems and fixed examples

Your name

Your institution

Meeting and date

What we will explore

Write your introduction here. Write formulas as usual, such as $f(z) = z^2$.

- Specify the mathematical object by a formula.

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- Specify the mathematical object by a formula.
- Manipulate a picture or game and compare the views.
- Relate your observations to a definition or criterion.

Let $\Omega \subset \mathbb{C}$ be open and $u\colon \Omega \to \mathbb{R}$ continuous.

Definition (Subharmonicity in the continuous case)

The function u is subharmonic if

$$u(a) \leq \frac{1}{2\pi} \int_0^{2\pi} u\big(a + re^{i\theta}\big)\, d\theta$$

for every closed disc $\overline{D(a,r)} \subset \Omega$, $r > 0$.

Definitions and theorems

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Theorem

If $u \in C^2(\Omega, \mathbb{R})$, then u is subharmonic if and only if $\Delta u = u_{xx} + u_{yy} \geq 0$ on Ω.

An example and a proof

Example

The function $u(x+iy) = 2x^2 - y^2$ is subharmonic on $\mathbb{C}$.

An example and a proof

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The function $u(x+iy) = 2x^2 - y^2$ is subharmonic on $\mathbb{C}$.

Proof.

Since $u_{xx} = 4$ and $u_{yy} = -2$,

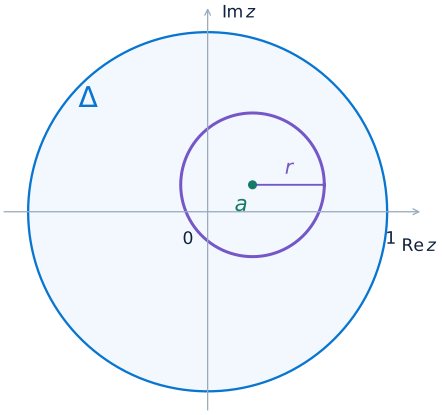
$$\Delta u = u_{xx} + u_{yy} = 2 > 0.$$

□

However, along the imaginary axis, $u(iy) = -y^2$.

Use the pictures to compare subharmonicity with convexity along real lines.

Insert an image



A circle with center a and radius r.

Explain the figure and what the audience should observe.

Use the same structure when replacing this picture.

A harmonic saddle

EXPLORE A FIXED EXAMPLE

Example ID: `sh / std-b03`

Explore this fixed example in the HTML presentation.
Switch between the question and the answer with explanation.

Public game (the fixed example appears in the HTML presentation)

What coordinate slices do not reveal

EXPLORE A FIXED EXAMPLE

Example ID: `psh / A03`

Explore this fixed example in the HTML presentation.
Switch between the question and the answer with explanation.

Public game (the fixed example appears in the HTML presentation)

Slices of a Hartogs figure

EXPLORE A FIXED EXAMPLE

Example ID: `ryokuinzu / n18`

Explore this fixed example in the HTML presentation.
Switch between the question and the answer with explanation.

Public game (the fixed example appears in the HTML presentation)

Conclusions and the next question

Write your conclusions or future questions here. Plain paragraphs are also welcome; bullet points are optional.

Question

Which definition or criterion explains the phenomenon we observed?

You may also list references and related websites here.